ON THE CHOICE OF THE OBJECTIVE FUNCTION AND OPTIMIZATION STRATEGY FOR METRIC DRIVEN MESH ADAPTION

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In this work we consider the problem of optimizing an unstructured three-dimensional grid in order to conform to a given Riemannian metric. The metric holds the information regarding the local size and local shape of the desired mesh, as first proposed in ref. [1], and can be either user-defined, or obtained from an a-posteriori error estimate.

At first, we study the problem of how to best define the objective function for the mesh optimization process. Our analysis starts from the definition of "compliance" of the actual metric to the given target. This leads to a matrix residual, here termed the "compliance residual". The compliance residual can be gauged by different "measure" functions, whose choice can have a non negligible effect on the overall effectivity of the procedures [2]. In this work we explore some of the different possible resulting objective functions, trying to highlight their merits and deficiencies, through the analysis of: a) their "shape" in the solution space, that should account for the fact that local mesh retriangulations only allow a limited number of possible discrete steps in the solution space; b) their sensitivity to all information contained in the metrics, so as to exercise the proper control on the shape and and on the size of the generated elements.

Next, we design an optimization process based on a Gauss-Seidel iterative scheme. The algorithm is based on the removal of elements that do not satisfy the chosen measure of the compliance residual to a given tolerance. The removal of each element can be attempted with several local retriangulation primitives, that either add vertices to the grid, remove vertices, retriangulate sets of vertices without changing their location or move vertices without altering their connectivities. Here again, as for the choice of the objective function, there is ample freedom in the way one can organize the process. In particular, one can either put a large effort in each element removal attempt, or one can try to reduce this effort, with the hope of reducing the overall computational cost of the optimization process. We design an algorithm that accommodates these different strategies in one single unified framework, and we test it on a number of representative examples. Numerical evidence leads to the conclusion that it is usually wise to invest the maximum possible effort in each single element removal attempt.

Numerical simulations are used throughout this work for studying the features of the various algorithms here tested, and for highlighting the most effective combinations of the various possible choices in the design of practical mesh optimization schemes.

References

[1] M.G. Vallet, *Génération de Maillages Éléments Finis Anisotropes et Adaptatifs*, Ph.D. Thesis, Université Pierre et Marie Curie, Paris VI, France, 1992.

[2] C.L. Bottasso, "Anisotropic mesh adaption by metric-driven optimization," *International Journal for Numerical Methods in Engineering*, under review, 2002.